Approximate Upper and Lower Bounds on the VSWR of a 3-Port Circulator with Nonideal Loads

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***Abstract*—The upper and lower bounds on the voltage standing wave ratio (VSWR) at port 1 of a 3-port circulator with a complex load under any phase conditions at port 2 and a nonideal termination at port 3 is of some interest in microwave engineering. The purpose of this letter is to give a solution to this problem under some simplifying conditions. The lower bound obtained here is in keeping with the definition of the complex gyrator circuit of this sort of junction when either port 2 or port 3 is terminated in its complex conjugate load.**

I. Introduction

THE voltage standing wave ratio (VSWR) at port 1 of a 3-port circulator is in practice not only dependent upon its specification, but also upon the load condition at port 2 and the specification of the termination at port 3. While an exact formulation of the problem is in principle available, it is too complicated for engineering practice [1]. In order to avoid this difficulty, a nomogram which allowed the worst case situation to be determined was introduced based on some simplifying conditions [2]. In obtaining this result the termination at port 3 was idealized and the VSWR of the circulator at port 2 was disregarded or absorbed into the antenna specification. Any reflection loss at port 1 was separately neglected in forming the incident wave at port 2. Its effect, however, was retained in calculating the effective VSWR at port 1.

Another approximation to this problem in which the effect of the load specification at port 3 is neglected but for which the circulator specification at port 2 is retained in outlined in [3]. Some related papers are given in [4]–[10]. The purpose of this contribution is to examine this problem under realistic conditions at ports 2 and 3. It includes statements about the upper and lower bounds on the effective VSWR at port 1.

II. Specification of 3-Port Circulators with Nondeal Loads

One solution to the problem in question has been derived by disregarding any reflection at port 1 in constructing the incident wave at port 2, by idealizing the circulator at port 2 and its termination at port 3. Any secondary reflections at port 1 have been separately neglected. The development introduced here retains the original assumption whereby the incident wave at port 2 is taken as unity instead of $\sqrt{1-\rho_{\mathrm{C}}^2}$ and whereby any secondary reflections at port 1 are neglected.

Manuscript received July 25, 1997. This work was supported by Apollo Microwave, Ltd., Montreal, Canada.

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Publisher Item Identifier S 1051-8207(97)08975-7.

Since practical components are usually described in terms of a VSWR specification this is the notation adopted here.

The required derivation begins with the rules governing the combinations of two discrete VSWR's S_1 and S_2 associated with two neighboring discontinuities. The exact upper and lower bounds on the resultant VSWR's are defined in terms of the individual ones in the usual way by

$$S_{\max} = S_1 S_2 \tag{1}$$

$$S_{\min} = S_1/S_2 \text{ or } S_2/S_1, \qquad S_{\min} \geq 1. \tag{2}$$

If the upper bound on the VSWR at port 1 is formulated to start with then the development begins by forming the worst case network parameters at port 2

$$S_{\max}^{(2)} = S_{\mathrm{C}} S_{\mathrm{A}} \tag{3}$$

$$\rho_{\max}^{(2)} = \frac{S_{\mathrm{C}} S_{\mathrm{A}} - 1}{S_{\mathrm{C}} S_{\mathrm{A}} + 1}. \tag{4}$$

The corresponding variables at port 3 are separately given by

$$S_{\max}^{(3)} = S_{\mathrm{C}} S_{\mathrm{L}} \tag{5}$$

$$\rho_{\max}^{(3)} = \frac{S_{\mathrm{C}} S_{\mathrm{L}} - 1}{S_{\mathrm{C}} S_{\mathrm{L}} + 1}. \tag{6}$$

S_{C} is the VSWR of the circulator at any port, S_{A} is that of the antenna at port 2, and S_{L} is that of the load at port 3. A schematic diagram of the arrangement considered here is indicated in Fig. 1.

The derivation now proceeds by forming the overall reflected or re-radiated wave produced at port 1 due to an incident wave at the same port

$$\rho_{\mathrm{refl}} = 1 \cdot \rho_{\max}^{(2)} \cdot \rho_{\max}^{(3)}. \tag{7}$$

The VSWR at the input port corresponding to this reflection coefficient is given by

$$S_{\mathrm{refl}} = \frac{1 + \rho_{\max}^{(2)} \rho_{\max}^{(3)}}{1 - \rho_{\max}^{(2)} \rho_{\max}^{(3)}}. \tag{8}$$

Writing this quantity in terms of the original variables gives

$$S_{\mathrm{refl}} = \frac{(S_{\mathrm{C}} S_{\mathrm{A}})(S_{\mathrm{C}} S_{\mathrm{L}}) + 1}{(S_{\mathrm{C}} S_{\mathrm{A}}) + (S_{\mathrm{C}} S_{\mathrm{L}})}. \tag{9}$$

The worst case effective VSWR at port 1 is now obtained by forming the product of the VSWR due to the reflected or re-radiated wave at port 1 and that of the circulator:

$$S_{\mathrm{eff}} = S_{\mathrm{C}} S_{\mathrm{refl}}. \tag{10}$$

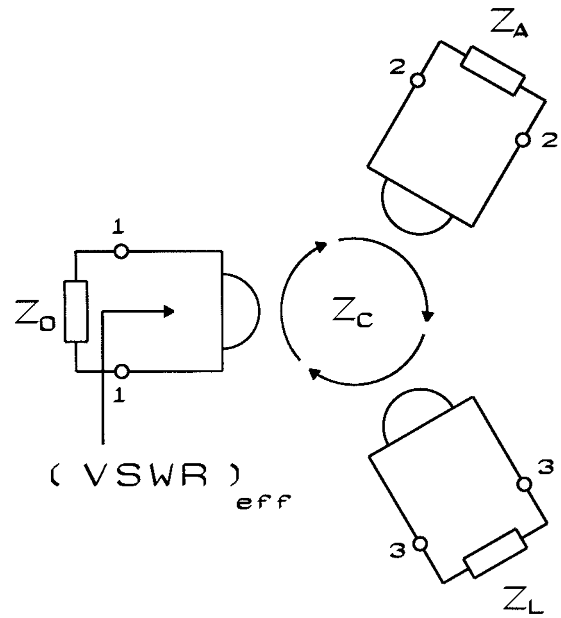

Fig. 1. Schematic diagram of 3-port junction circulator with nonideal loads.

The required result is

$$S_{\text{eff}} = \frac{(S_C S_A)(S_C S_L) + 1}{S_A + S_L}. \tag{11}$$

This equation may also be readily solved for either $S_A S_C$ or S_L in terms of S_{eff}

$$S_A = \frac{-1 + S_{\text{eff}} S_L}{-S_{\text{eff}} + S_C^2 S_L} \tag{12}$$

$$S_L = \frac{-1 + S_A S_{\text{eff}}}{-S_{\text{eff}} + S_C^2 S_A} \tag{13}$$

and

$$S_C = \left[\frac{S_{\text{eff}}(S_A + S_L) - 1}{S_A S_L}\right]^{1/2}. \tag{14}$$

The derivation of the best possible situation at port 1 may be deduced without ado by replacing $S_{\max}^{(2)}$ and $S_{\max}^{(3)}$ by

$$S_{\min}^{(2)} = S_A / S_C, \qquad S_A \geq S_C \tag{15}$$

$$S_{\min}^{(3)} = S_C / S_L, \qquad S_C \geq S_L \tag{16}$$

and forming the ratio S_{refl}/S_C instead of the product $S_{\text{refl}} S_C$. Taking the effective VSWR, that of the circulator and those of the load and termination's one at time as the dependant variable gives

$$S_{\text{eff}} = \frac{S_C^2 + S_A S_L}{S_A + S_L}, \qquad S_{\text{refl}} \geq S_C \tag{17}$$

$$S_A = \frac{S_C^2 - S_L S_{\text{eff}}}{S_{\text{eff}} - S_L}, \qquad S_{\text{refl}} \geq S_C \tag{18}$$

$$S_L = \frac{S_A S_{\text{eff}} - S_C^2}{S_A - S_{\text{eff}}}, \qquad S_{\text{refl}} \geq S_C \tag{19}$$

$$S_C = [S_{\text{eff}}(S_A + S_L) - S_A S_L]^{1/2}, \qquad S_{\text{refl}} \geq S_C. \tag{20}$$

Fig. 2 indicates the relationship between the best and worst bounds on the VSWR at port 1 and the antenna load at port 2 for two different circulator specifications and one typical

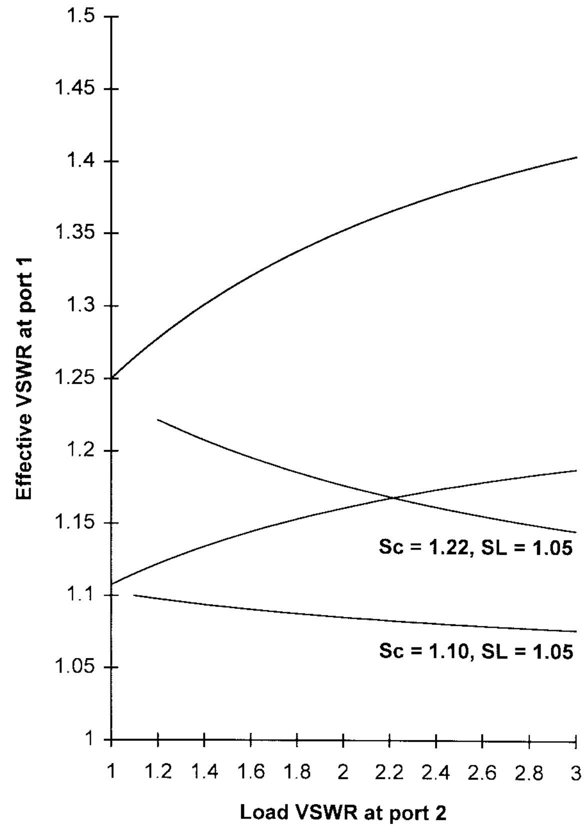

Fig. 2. Upper and lower bounds on effective VSWR of load at port 1 versus VSWR at port 2 for different circulator and termination specifications.

load specification. The lower bounds in these curves are left blank between the origin and $S_A \leq S_C$ in keeping with the condition in (15). If either S_A or S_L equals S_C then S_{eff} in (17) reduces to

$$S_{\text{eff}} = S_C. \tag{21}$$

These two load conditions coincide, of course, with either that for which Z_A or Z_L is the complex conjugate of Z_C. These are in keeping with the definition of the gyrator circuit of this type of junction.

Taking the particular case for which $S_A = S_L = S_C = 1.22$ indicates that S_{eff} is bracketed between 1.22 and 1.32.

III. Conclusions

The reflection coefficient at port 1 of a 3-port circulator is in general made up of a component associated with the incident wave at port 1 and another due to the re-radiated wave at the same port. Simple approximate closed-form expressions for the upper and lower bounds on the VSWR at port 1 of a 3-port circulator with ports 2 and 3 terminated in arbitrary loads have been derived. The robustness of the approximations utilized in this work has been verified by introducing either the condition $S_A = S_C$ or $S_L = S_C$ in the lower bound relationship. A scrutiny of each case one at a time indicates that S_{eff} equals S_C in keeping with the exact result.

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